Alpen Adria Universität Klagenfurt
Institut für Statistik

H. Kazianka **No. 5**

Exercises "VU Finanzstatistik (SS 2014)"

25. Prove that the price process $(X_t)_{t\in[0,T]}$ has an arbitrage if and only if the normalized price process $(\bar{X}_t)_{t\in[0,T]}$ has an arbitrage.

26. Suppose the price process $(X_t)_{t\in[0,T]}$ is normalized. Prove that $(X_t)_{t\in[0,T]}$ has an arbitrage if and only if there exsists an admissible portfolio θ_t such that

$$V_0^\theta \leq V_T^\theta \text{ a.s. and } \mathbb{P}[V_T^\theta > V_0^\theta] > 0.$$

Hint: Define $\tilde{\theta}_t = (\tilde{\theta}_t^0, \tilde{\theta}_t^1, \ldots, \tilde{\theta}_t^n)^T$ *as follows:* $\tilde{\theta}_t^i = \theta_t^i$ *for* $i = 1, \ldots, n$, $\tilde{\theta}_0^0$ *such that* $V_0^{\tilde{\theta}} = 0$ *and* $\tilde{\theta}_t^0$ *to make* $\tilde{\theta}$ *self-financing. Show that* $V_t^{\tilde{\theta}} = V_t^\theta - V_0^\theta$ *and draw the conclusions.*

27. Let $\theta_t = (\theta^0, \theta^1, \ldots, \theta^n)^T$ be a constant portfolio. Show that θ_t is self-financing.

28. Check which of the following normalized markets allow an arbitrage. If so, find one.

   a) $dS_t^1 = dt + dB_t^1 + dB_t^2 - dB_t^3$, $dS_t^2 = 5dt - dB_t^1 + dB_t^2 + dB_t^3$.

   b) $dS_t^1 = dt + dB_t^1 + dB_t^2 - dB_t^3$, $dS_t^2 = 5dt - dB_t^1 - dB_t^2 + dB_t^3$.

   c) $dS_t^1 = dt + dB_t^1 + dB_t^2$, $dS_t^2 = 2dt + dB_t^1 - dB_t^2$, $dS_t^3 = -2dt - dB_t^1 + dB_t^2$.