Alpen Adria Universität Klagenfurt
Institut für Statistik

H. Kazianka **No. 4**

Exercises "VU Finanzstatistik" (SS 2014)

21. Solve the following stochastic differential equation with starting value $(X_0^1, X_0^1)^T = (0,0)^T$:

$$\begin{pmatrix} dX_t^1 \\ dX_t^2 \end{pmatrix} = \begin{pmatrix} 1 \\ X_t^2 \end{pmatrix} dt + \begin{pmatrix} 0 \\ e^{X_t^1} \end{pmatrix} dB_t.$$

22. Solve the following stochastic differential equations:

    a) $dX_t = tX_t dt + e^{t^2/2} dB_t$ with $X_0 = 1$,

    b) $dX_t = -\frac{1}{1+t} X_t dt + \frac{1}{1+t} dB_t$ with $X_0 = 0$.

23. Let

$$dY_t = \begin{pmatrix} 0 \\ 1 \end{pmatrix} dt + \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} dB_t^1 \\ dB_t^2 \end{pmatrix}, \quad t \in [0,T],\ T > 0.$$

Use Girsanov's Theorem (version II) to find a probability measure $\mathbb{Q}$ on $\mathcal{F}_T$ such that

$$dY_t = \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} d\tilde{B}_t^1 \\ d\tilde{B}_t^2 \end{pmatrix},$$

where $(\tilde{B}_t^1, \tilde{B}_t^2)^T$ is 2-dimensional Brownian motion with respect to $\mathbb{Q}$. How is $(\tilde{B}_t^1, \tilde{B}_t^2)^T$ related to $(B_t^1, B_t^2)^T$, the Brownian motion with respect to $\mathbb{P}$?

24. Let $dY_t = dB_t$ with $Y_0 = x$, $x \in \mathbb{R}$ for $t \in [0,T]$. Show with the help of Girsanov's Theorem (version III) that $(Y_t, \tilde{B}_t)$ is a weak solution in $[0,T]$ to the stochastic differential equation $dX_t = a(X_t)dt + dB_t$ with $X_0 = x$ if $\tilde{B}_t = B_t - \int_0^t a(x + B_s)ds$ is a Brownian motion with respect to $\mathbb{Q}$, where $\mathbb{Q}$ is defined as $d\mathbb{Q} = M_T d\mathbb{P}$ on $\mathcal{F}_T$ with $M_t = \exp\left\{\int_0^t a(x+B_s)dB_s - \frac{1}{2}\int_0^t |a(x+B_s)|^2 ds\right\}$.