Non-linear control for energy efficient DC-DC converters supporting DCM operation

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Abstract—Non-linear control is a popular way to regulate the output voltage of switch-mode DC-DC converters. In this paper, a controller based on the non-linear sliding mode theory, but adapted to maintain a constant switching frequency, is proposed. A general definition of the sliding surface is suggested, making it possible to use the same control architecture for different DC-DC converter topologies (e.g. Buck, Boost, Buck-Boost).

Moreover, the proposed controller supports the discontinuous conducting mode (DCM) to increase the efficiency of the energy conversion under light loads. Several simulation results are presented in order to prove the performance of the controller. A real-world commercial application, i.e. an efficient power supply for mobile devices, has been used as a reference to accurately choose the simulation parameters.

I. INTRODUCTION

Energy efficiency is an aspect of key importance in the design of a controller for a pulse-width modulation (PWM) DC-DC converter. This is especially true when the application is a portable device (e.g. PDAs, mobile phones) where battery lifetime is a critical design parameter. A key to maximize the efficiency of the conversion is the choice of an appropriate controller structure. For example, a controller minimizing the output voltage undershoot during load jumps allows a decrement of the DC value of the reference voltage, reducing the static power consumption. The efficiency can also be increased by utilizing the discontinuous conduction mode (DCM) when a light load is applied. A lot of different controller architectures can be employed to achieve these tasks. While the most popular choice arguably is a linear proportional-integral-derivative (PID) controller [1], [2], several controllers based on the sliding mode theory have been proposed [3]–[5]. Sliding mode controllers are known to be more robust to line, load and parameter variations [6], [7] and often provide better dynamic performance with respect to PID controllers.

Another important aspect is the switching frequency \( f_{\text{sw}} \) at which the converter operates. In a mobile application it is often desirable to keep \( f_{\text{sw}} \) constant, in order to avoid interference with other parts of the system, such as the RF circuitry. However, a traditional sliding mode controller inherently operates at a variable frequency, therefore requiring additional care to keep \( f_{\text{sw}} \) constant. Several techniques have been presented [8] for this purpose. In this paper, a fixed-frequency sliding mode controller for a DC-DC converter is illustrated. The controller is capable to operate the converter both in continuous conduction mode (CCM) and in DCM. The proposed controller differs from previous works [5], [9], since no hysteretic comparator is used and no parameters have to be changed when switching between DCM and CCM. Moreover, the adopted definition of the sliding surface allows to use the same controller for different converter topologies (e.g. Buck, Boost, Buck-Boost). Simulation results are shown to verify the effectiveness of the controller to regulate both a step-down (Buck) and a step-up (Boost) converter.

II. SLIDING MODE CONTROL

Due to the fact that DC-DC converters are inherently variable structure systems (VSS) (i.e. their topology changes during operation), they can be considered as a very well-suited application for the sliding mode theory [6]. A converter can be modeled with the following general expression:

\[
\dot{x} = f(x, t, u),
\]

where \( x \in \mathbb{R}^n \) is the state-variables vector, \( t \in \mathbb{R} \) is time and \( u \in \{u^+, u^-\} \) is the PWM signal that drives the switches of the power stage. Since the structure of the converter changes over time, the function \( f \) is discontinuous on a surface \( \sigma(x, t) = 0 \) and can be written as:

\[
f(x, t, u) = \begin{cases} 
   f^+(x, t, u^+) & \text{if } \sigma \rightarrow 0^+ \\
   f^-(x, t, u^-) & \text{if } \sigma \rightarrow 0^-.
\end{cases}
\]

The action of the sliding mode controller is to force the representative point (RP) of the plant in the phase plane to stay (or slide) on the surface \( \sigma(x, t) = 0 \) (also called sliding surface) until it reaches the desired equilibrium point. The controller ensures that the RP stays on the sliding surface by applying the following control law:

\[
u = \begin{cases} 
   u^+ & \text{if } \sigma(x) > 0 \\
   u^- & \text{if } \sigma(x) < 0.
\end{cases}
\]

The sliding surface must be chosen carefully, ensuring that the existence, reachability and stability conditions are met [6]. Throughout the rest of the paper, the following definition of the sliding surface will be used:

\[
\sigma(x, t) = s^T x(t) = s_1 x_1(t) + s_2 x_2(t) + s_3 x_3(t),
\]

where the first sliding coefficient \( s_1 \) is assumed to be equal to one without loss of generality, since the controller forces
the sliding surface to be $\sigma(x, t) = 0$. The feedback variables vector $x$ has been chosen as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_o - V_{\text{ref}} \\ i_L \\ \int (v_o - V_{\text{ref}}) \, dt \end{bmatrix}, \quad (5)$$

where $v_o$ is the output voltage of the DC-DC converter, $V_{\text{ref}}$ is the reference voltage and $i_L$ is the inductor current. The advantage of using the coil current $i_L$ instead of the time derivative of the output voltage $v_o$ in the sliding surface definition lies in the fact that the same controller can be applied to different converter topologies, e.g. a Boost converter where $v_o$ is not continuous and, as such, cannot be employed in the definition of the sliding surface. The drawback is that the inductor current $i_L$ must be sensed.

### A. Application to a Buck converter

Assuming that the feedback variables vector $x$ is defined as in (5), the state-space representation of a Buck converter, which is schematically reported in Fig. 1, operating in CCM as in (5), the state-space representation of a Buck converter, can be written as:

$$\dot{x} = Ax + Bu + D,$$

$$A = \begin{pmatrix} -\frac{1}{R_C C} & -\frac{1}{C} & 0 \\ -\frac{1}{L} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \frac{V_{\text{in}}}{L} \\ 0 \end{pmatrix}, \quad D = \begin{pmatrix} -\frac{V_{\text{ref}}}{R_C} \\ \frac{V_{\text{ref}}}{L} \\ 0 \end{pmatrix},$$

where $L$ and $C$ are the values of the inductance and capacitance of the output filter, $V_{\text{in}}$ is the input (battery) voltage and $R_i$ is the load resistance. In order to check the existence conditions of the sliding mode, we introduce a Lyapunov function $V(x)$, defined as:

$$V(x) = \frac{1}{2} \sigma^T(x) \sigma(x),$$

which is positive definite. Then we must ensure that its time derivative is negative definite:

$$\frac{dV(x)}{dt} = \sigma^T \frac{d\sigma}{dt} = \sigma (\nabla \sigma \cdot f) = \sigma (s^T \cdot f) < 0. \quad (6)$$

In the case of the Buck converter, it can be derived [5] that these relations reduce to the following conditions:

$$\begin{cases} \frac{1}{C} i_L + \frac{V_{\text{in}} - V_{\text{ref}}}{L} = \frac{V_{\text{ref}}}{R_C} > 0 & \text{if } \sigma < 0 \\ -\frac{1}{C} i_L - \frac{V_{\text{ref}}}{L} = \frac{V_{\text{ref}}}{R_C} < 0 & \text{if } \sigma > 0, \end{cases}$$

It is trivial to check that the reachability conditions are met since the steady-state RP corresponding to $u = u^+$ lies in the portion of the state space corresponding to $u = u^-$ and vice versa.

The last condition that has to be verified is the stability of the system under sliding mode operation. As soon as the RP reaches the sliding surface, it will continue to stay there, i.e. $\dot{\sigma} = 0$, yielding:

$$\frac{dx_1}{dt} + s_2 \frac{dx_2}{dt} + s_3 x_1 = 0$$

which can be reduced to a differential equation in $x_1$ (i.e. the error on the output voltage) only:

$$\frac{d^2 x_1}{dt^2} + \frac{1}{C s_2} \left(1 + \frac{s_2}{R_C} \right) \frac{dx_1}{dt} + \frac{s_3}{C s_2} x_1 = 0 \quad (7)$$

From (7) we obtain that, in order to meet the stability conditions, both sliding coefficients $s_2$ and $s_3$ must be positive. Furthermore, a simple description of the dynamic behavior of the system can be obtained, by observing that the system under sliding-mode is equivalent to a second-order system with a natural frequency $\omega_n$ equal to

$$\omega_n = \sqrt{\frac{s_3}{s_2 C}}. \quad (8)$$

For typical values of the sliding coefficients, we have $s_2/R_C \ll 1$ and a simple expression of the damping factor $\xi$ can be derived:

$$\xi \approx \frac{1}{2 \sqrt{s_2 s_3 C}} \quad (9)$$

With the aid of (8) and (9) one can choose the values of the sliding coefficients in order to meet the desired dynamic performance of the system.

### B. Constant switching frequency

In many applications, such as a portable device (e.g. a mobile phone), an important constraint on the operation of the converter rises from the fact that it is highly desirable that the switching frequency $f_{sw}$ remains constant. This is...
usually ensured in order to avoid interference with other parts of the system, such as the RF circuitry. Nevertheless, a small fluctuation of $f_{sw}$ might be allowed for a short time span, e.g., during a load jump.

The “conventional” sliding mode control operates at a variable switching frequency and thus needs some modifications. Several methods have been proposed in literature, such as dynamically adapting the amplitude of the hysteresis band [8], [10] or by using a PWM-based sliding mode approach [4]. In this paper, a different technique is used: the PWM signal $u$ is set to a logical “1” at every rising edge of a clock signal (running at the desired switching frequency $f_{sw}$) and then it is reset to 0 when the sliding function reaches the value $\sigma(x) = 0$. A schematic representation of the controller is reported in Fig. 2. It is worth noting that, with this architecture, there could be short fluctuations of the switching frequency $f_{sw}$ during load jumps, because two consecutive positive edges of the clock signal could occur without a reset event.

C. DCM operation

The proposed controller has been adapted to support DCM operation without changing the sliding coefficients or altering the control algorithm. It is sufficient to detect the zero-crossings of the inductor current $i_L$ and turn off both the high-side and low-side switches of the power stage.

A major advantage over a linear PID controller lies in the fact that the sliding mode controller is robust to large variations on line, load and parameter variations without modifying the sliding coefficients when a transition from DCM to CCM (and vice versa) occurs. A PID controller, on the other hand, would need a reconfiguration of the controller coefficients to operate in an optimal fashion in both modes. This is due to the fact that the equivalent transfer function of the power converter changes when switching between operating modes, and a linear controller is optimized around a pre-determined operating point.

III. SIMULATION RESULTS

A detailed model of the system has been developed in order to provide reliable results [11]. Various parasitic components of the power stage have been included in the model, such as the equivalent series resistance (ESR) and equivalent series inductance (ESL) of the output capacitor ($R_C$ and $L_C$ respectively), the direct-current resistance (DCR) $R_L$ of the coil, the equivalent on-resistances $R_p$ and $R_n$ (for the p-MOSFET and n-MOSFET, respectively), parasitic capacitances and diodes of the MOSFETs. A schematic representation of the converter is reported in Fig. 1. Realistic values of the parameters have been used for simulations, using a real-world commercial application [12] as a reference. The numerical value of the parameters of the converter are reported in Table I. The sliding coefficients have been tuned in order to yield $\xi \approx 1$ and an adequate equivalent bandwidth $\omega_n$. The numerical values of the coefficients are reported in Table II.

Fig. 3 shows the output voltage $v_o$ and inductor current $i_L$ versus time, when the load current varies from 20 mA to 600 mA and then back to 20 mA, requiring transitions between CCM and DCM. It can be seen that the values of the undershoot and overshoot are approximately equal to 20 mV which is 1.1% of the reference voltage. Fig. 4 is a zoomed version of Fig. 3 around the time instant when the positive load jump occurs. From the reported graph, it is clearly visible that the controller efficiently manages the transition from DCM to CCM mode at the load jump instant. In fact, the new load current value is almost reached in one switching period, starting from DCM operation.

Extension to other converter topologies: As stated earlier, the application of the same controller scheme can be extended to other converter topologies. In this section, simulation results are shown for a Boost converter. The system parameters are essentially the same as reported in Table I with the only difference that the values of the input and reference voltages are now set to $V_{in} = 3$ V and $V_{ref} = 3.5$ V, respectively. The sliding coefficients have also been adapted to optimize the transient response, and their values are reported in Table II.

![Fig. 3. Output voltage $v_o$ and inductor current $i_L$ waveforms of a transient response of a Buck converter to a load jump from 20 mA to 600 mA at $t = 0.55$ ms and then back to 20 mA at $t = 0.65$ ms.](image-url)
Fig. 4. Output voltage $v_o$ and inductor current $i_L$ waveforms around the load jump instant $t = 0.55$ ms.

Fig. 5. Output voltage $v_o$ and inductor current $i_L$ waveforms of a transient response of a Boost converter to a load jump from 20 mA to 600 mA at $t = 0.6$ ms and then back to 20 mA at $t = 0.7$ ms.

Fig. 6. Output voltage $v_o$ and inductor current $i_L$ waveforms around the load jump instant $t = 0.6$ ms.

Fig. 5 shows the output voltage $v_o$ and inductor current $i_L$ versus time, when the load current varies from 20 mA to 600 mA and then back to 20 mA. It is worth noting that, for a Boost converter, the average value of the coil current is higher than the load current DC value since the load is not always connected to the inductor [1]. Fig. 6 is a zoomed version of Fig. 5 around the time instant when the positive load jump occurs. Again, it can be seen that the controller is capable of managing the transition from DCM to CCM mode.

IV. CONCLUSIONS

A non-linear controller for switch-mode DC-DC converters based on the sliding mode theory has been presented. The control architecture has been slightly modified with respect to a conventional sliding mode controller in order to achieve a constant operating frequency. The controller has been successfully extended in order to support DCM operation. The adopted sliding surface definition allows the application of the proposed controller scheme to different DC-DC converter topologies. Applications to a Buck and a Boost converter have been presented with simulation results proving the capabilities of the proposed scheme.

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